

TECHNICAL DATA

The moment of inertia of a planar figure with respect to a generic point, with Cartesian axes having their origin always in that point, is equal to the sum of the moments of inertia of each individual quadrant (always positive in x and y), hypothetically decomposed from the original Cartesian system.

Therefore, in the method of "Cartesian Decomposition", the total moment of inertia is equal to the sum of the moments of inertia of each quadrant. See "Proof of the method.ENG" on the website.

$$I_{x1} = \sum_{j,i=1}^n m_{j,i} d_{y,j,i}^2 = m_{1,1} d_{y1,1}^2$$

$$I_{x2} = \sum_{j,i=1}^n m_{j,i} d_{y,j,i}^2 = m_{2,2} d_{y2,2}^2$$

$$I_{x3} = \sum_{j,i=1}^n m_{j,i} d_{y,j,i}^2 = m_{3,3} d_{y3,3}^2$$

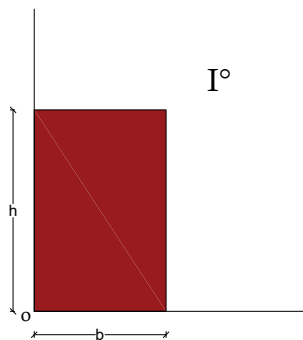
$$I_{x4} = \sum_{j,i=1}^n m_{j,i} d_{y,j,i}^2 = m_{4,4} d_{y4,4}^2$$

$$I_x = \sum_{j,i=1}^n I_{xj} = I_{x1} + I_{x2} + I_{x3} + I_{x4} = \sum_{j,i=1}^n m_{j,i} d_{y,j,i}^2$$

$$I_x = m_{1,1} d_{y1,1}^2 + m_{2,2} d_{y2,2}^2 + m_{3,3} d_{y3,3}^2 + m_{4,4} d_{y4,4}^2 = m_1 d_{y1}^2 + m_2 d_{y2}^2 + m_3 d_{y3}^2 + m_4 d_{y4}^2$$

$$I_y = \sum_{j,i=1}^n I_{yj} = I_{y1} + I_{y2} + I_{y3} + I_{y4} = \sum_{j,i=1}^n m_{j,i} d_{x,j,i}^2$$

$$I_y = m_{1,1} d_{x1,1}^2 + m_{2,2} d_{x2,2}^2 + m_{3,3} d_{x3,3}^2 + m_{4,4} d_{x4,4}^2 = m_1 d_{x1}^2 + m_2 d_{x2}^2 + m_3 d_{x3}^2 + m_4 d_{x4}^2$$



$$A = b \cdot h$$

$$x_g = b / 2 \quad y_g = h / 2$$

$$S_x = A / 2 \cdot y_g / 2 \quad S_y = A / 2 \cdot x_g / 2$$

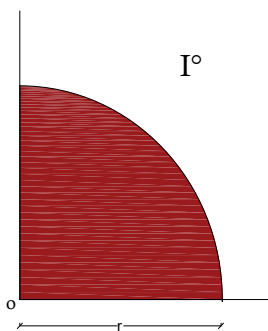
$$I_{o,x} = b \cdot h^3 / 3 \quad I_{o,y} = b^3 \cdot h / 3$$

$$w_{pl,x} = S_x \cdot 2 \quad w_{pl,y} = S_y \cdot 2$$

$$I_{x,g} = b \cdot h^3 / 12 \quad I_{y,g} = h \cdot b^3 / 12$$

$$w_{el,x} = I_{x,g} / y_g \quad w_{el,y} = I_{y,g} / x_g$$

$$r_{x,g} = I_{x,g} / A \quad r_{y,g} = I_{y,g} / A$$



$$A = \pi \cdot r^2$$

$$y_g, x_g = \frac{2 \cdot r \cdot \sin(\alpha^\circ) \frac{180}{\pi} \cdot \cos(\vartheta)}{3\alpha}$$

$$S_x = A(y) / 2 \cdot y_g / 2 \quad S_y = A(x) / 2 \cdot x_g / 2$$

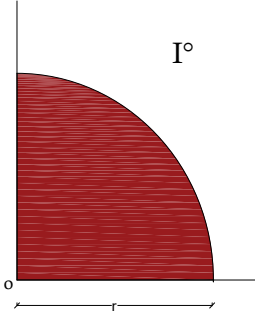
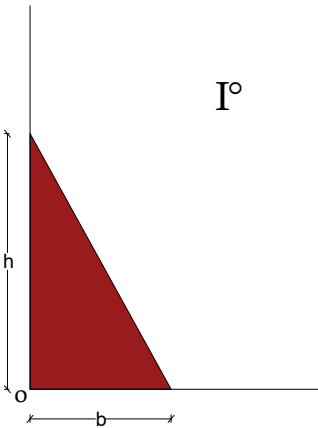
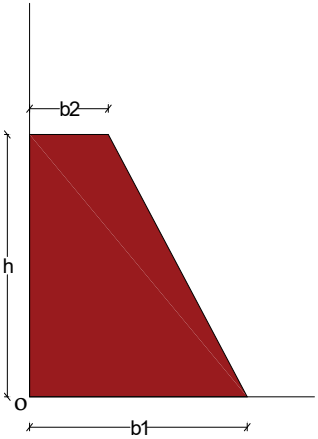
$$A(x) = A(y) = \left[\frac{R^2 \cdot \text{Arcsen}(x/r)}{2} + \frac{r \cdot x \cdot \sqrt{r^2 - x^2}}{2 \cdot r} \right]_0^r$$

$$x_g = \frac{1}{A(x)} \int \left(\int_0^{\sqrt{r^2 - x^2}} x \cdot dx \right) dy$$

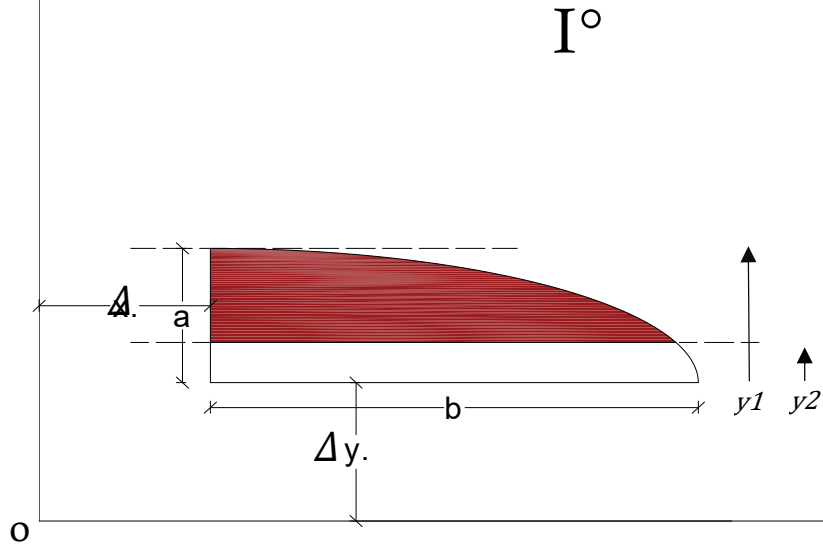
$$I_{o,x} = \pi \cdot D^4 / 256 \quad I_{o,y} = \pi \cdot D^4 / 256$$



TECHNICAL DATA

	$w_{pl,x} = S_x \cdot 2 \quad w_{pl,y} = S_y \cdot 2$ $\left[I_{x,g}; I_{y,g} \right] = \left[Steiner - hyugens \left(I_{o,x}; I_{o,y} \right) \right]$ $w_{el,x} = I_{x,g} / y_g \quad w_{el,y} = I_{y,g} / x_g$ $r_{x,g} = I_{x,g} / A \quad r_{y,g} = I_{y,g} / A$
	$A = b \cdot h / 2$ $x_g = b / 3 \quad y_g = h / 3$ $S_x = b_i \cdot h_i \cdot h_i / 3 \quad S_y = b_i \cdot h_i \cdot b_i / 3$ $I_{o,x} = b \cdot h^3 / 12 \quad I_{o,y} = b^3 \cdot h / 12$ $w_{pl,x} = S_x \cdot 2 \quad w_{pl,y} = S_y \cdot 2$ $I_{x,g} = b \cdot h^3 / 36 \quad I_{y,g} = h \cdot b^3 / 36$ $w_{el,x} = I_{x,g} / y_g \quad w_{el,y} = I_{y,g} / x_g$ $r_{x,g} = I_{x,g} / A \quad r_{y,g} = I_{y,g} / A$
	$A = (b_1 + b_2) \cdot h / 2$ $x_g = \frac{(b_1^2 + b_2^2 + b_1 \cdot b_2)}{3 \cdot (b_1 + b_2)} \quad y_g = \frac{h \cdot (2 \cdot b_2 + b_1)}{3 \cdot (b_1 + b_2)}$ $S_x = (b_{1,i} + b_{2,i}) \cdot h_i / 2 \cdot \frac{h_i \cdot (2 \cdot b_{2,i} + b_{1,i})}{3 \cdot (b_{1,i} + b_{2,i})} \quad S_y = (b_{1,i} + b_{2,i}) \cdot h_i / 2 \cdot \frac{(b_{1,i}^2 + b_{2,i}^2 + b_{1,i} \cdot b_{2,i})}{3 \cdot (b_{1,i} + b_{2,i})}$ $I_{o,x} = (b_1 - b_2) \cdot h^3 / 12 + b_2 \cdot h^3 / 3 \quad b_1 > b_2$ $I_{o,x} = (b_2 - b_1) \cdot h^3 / 12 + b_1 \cdot h^3 / 3 \quad b_1 < b_2$ $I_{o,y} = b_2 \cdot h^3 / 3 + h \cdot (b_1 - b_2)^3 / 36 + \left\{ \left[(b_1 - b_2) \cdot h / 2 \cdot \left[b_2 + (b_1 - b_2) / 3 \right]^2 \right] \right\} \quad b_1 > b_2$ $I_{o,y} = b_1 \cdot h^3 / 3 + h \cdot (b_2 - b_1)^3 / 36 + \left\{ \left[(b_2 - b_1) \cdot h / 2 \cdot \left[b_1 + (b_2 - b_1) / 3 \right]^2 \right] \right\} \quad b_1 < b_2$ $w_{pl,x} = S_x \cdot 2 \quad w_{pl,y} = S_y \cdot 2$ $\left[I_{x,g}; I_{y,g} \right] = \left[Steiner - hyugens \left(I_{o,x}; I_{o,y} \right) \right]$ $w_{el,x} = I_{x,g} / y_g \quad w_{el,y} = I_{y,g} / x_g$ $r_{x,g} = I_{x,g} / A \quad r_{y,g} = I_{y,g} / A$





$$\Delta A(y) = \frac{4b}{a} \int_0^a \sqrt{a^2 - y_1^2} dy - \frac{4b}{a} \int_0^a \sqrt{a^2 - y_2^2} dy$$

$$y_g = \frac{1}{A(y)} \iint y dy dx \qquad y_g = \frac{b}{A(y)} \int_0^{y_i} \sqrt{y_i^2 \left(1 - \frac{y_i^2}{a^2}\right)} dy$$

$$x_g = \frac{1}{A(y)} \iint y dy dx \quad x_g = \frac{1}{2 \cdot A(y)} \int_0^{y_i} b^2 \left(1 - \frac{y_i^2}{a^2}\right) dy$$

$$\Delta y_g = \frac{A(y)_1 \cdot y_{g,1} - A(y)_2 \cdot y_{g,2}}{A(y)_1 - A(y)_2} \quad \Delta x_g = \frac{1}{2 \cdot \Delta A(y)} \cdot \left[\int_0^{y_2} b^2 \left(1 - \frac{y_2^2}{a^2} \right) dy - \int_0^{y_1} b^2 \left(1 - \frac{y_1^2}{a^2} \right) dy \right]$$

$$\Delta I_y(x) = \iint x_1^2 dA - \iint x_2^2 dA = \iint x_1^2 dx dy - \iint x_2^2 dx dy \quad \Delta I_y = \int_0^{y_1} dy \int_0^{b/a\sqrt{a^2-y_1^2}} x_1^2 dx - \int_0^{y_2} dy \int_0^{b/a\sqrt{a^2-y_2^2}} x_2^2 dx$$

$$\Delta I_y(x) = \frac{b^3}{3 \cdot a^3} \int_0^{y_1} \sqrt{a^2 - y_1^2} dy - \frac{b^3}{3 \cdot a^3} \int_0^{y_2} \sqrt{b^2 - y_2^2} dy$$

$$\Delta I_x(x) = \iint y_1^2 dA - \iint y_2^2 dA = \iint y_1^2 dx dy - \iint y_2^2 dx dy \quad \Delta I_x = \int_0^{y_1} y_1^2 dy \int_0^{\sqrt{\left(1 - \frac{y_1^2}{a^2}\right) \cdot b^2}} dx - \int_0^{y_2} y_2^2 dy \int_0^{\sqrt{\left(1 - \frac{y_2^2}{a^2}\right) \cdot b^2}} dx$$

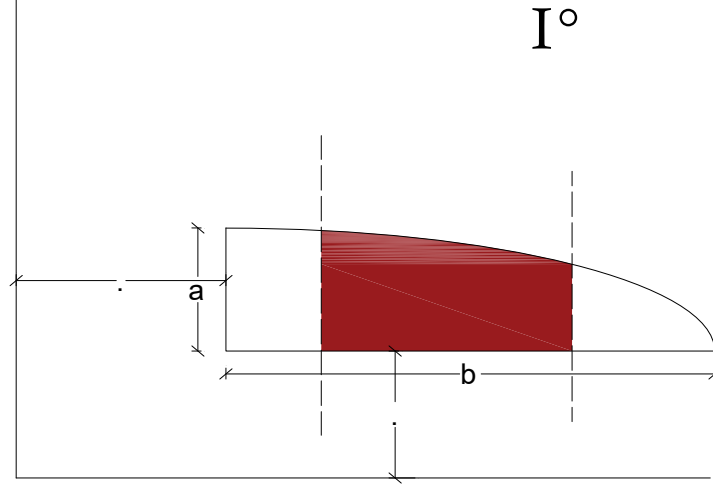
$$\Delta I_x(x) = \frac{b}{a} \int_0^{y_1} y_1^2 \sqrt{a^2 - y_1^2} dy - \frac{b}{a} \int_0^{y_2} y_2^2 \sqrt{a^2 - y_2^2} dy$$

$$\left[I_{x,g}; I_{y,g} \right] = \left[Steiner-hyugens(I_{o,x}; I_{o,y}) \right]$$

$$w_{el,x} = I_{x,g} / y_g \quad w_{el,y} = I_{y,g} / x_g$$

$$r_{x,g} = I_{x,g} / A \quad r_{y,g} = I_{y,g} / A$$





$$\Delta A(x) = \frac{4a}{b} \int_0^b \sqrt{b^2 - x_1^2} dx - \frac{4a}{b} \int_0^b \sqrt{b^2 - x_2^2} dx$$

$$x_g = \frac{1}{A(x)} \iint x dy dx \quad x_g = \frac{a}{A(x)} \int_0^{x_i} \sqrt{x_i^2 \left(1 - \frac{x_i^2}{b^2}\right)} dx$$

$$y_g = \frac{1}{A(x)} \iint y dy dx \quad y_g = \frac{1}{2 \cdot A(x)} \int_0^{x_i} a^2 \left(1 - \frac{x_i^2}{b^2}\right) dx$$

$$\Delta x_g = \frac{A(x)_2 \cdot x_{g,2} - A(x)_1 \cdot x_{g,1}}{A(x)_2 - A(x)_1} \quad \Delta y_g = \frac{1}{2 \cdot \Delta A(x)} \cdot \left[\int_0^{x_2} a^2 \left(1 - \frac{x_2^2}{b^2}\right) dx - \int_0^{x_1} a^2 \left(1 - \frac{x_1^2}{b^2}\right) dx \right]$$

$$\Delta I_x(x) = \iint y_1^2 dA - \iint y_2^2 dA = \iint y_1^2 dx dy - \iint y_2^2 dx dy \quad \Delta I_x = \int_0^{x_1} dx \int_0^{a/b\sqrt{b^2-x_1^2}} y_1^2 dy - \int_0^{x_1} dx \int_0^{a/b\sqrt{b^2-x_2^2}} y_2^2 dy$$

$$\Delta I_x(x) = \frac{a^3}{3 \cdot b^3} \int_0^{x_1} \sqrt{b^2 - x_1^2} dx - \frac{a^3}{3 \cdot b^3} \int_0^{x_2} \sqrt{b^2 - x_2^2} dx$$

$$\Delta I_y(x) = \iint x_1^2 dA - \iint x_2^2 dA = \iint x_1^2 dx dy - \iint x_2^2 dx dy \quad \Delta I_y = \int_0^{x_1} x_1^2 dx \int_0^{\sqrt{\left(1 - \frac{x_1^2}{b^2}\right)} \cdot a^2} dy - \int_0^{x_2} x_2^2 dx \int_0^{\sqrt{\left(1 - \frac{x_2^2}{b^2}\right)} \cdot a^2} dy$$

$$\Delta I_y(x) = \frac{a}{b} \int_0^{x_1} x_1^2 \sqrt{b^2 - x_1^2} dx - \frac{a}{b} \int_0^{x_2} x_2^2 \sqrt{b^2 - x_2^2} dx$$

$$[I_{x,g}; I_{y,g}] = [\text{Steiner-huygens}(I_{o,x}; I_{o,y})]$$

$$w_{el,x} = I_{x,g} / y_g \quad w_{el,y} = I_{y,g} / x_g$$

$$r_{x,g} = I_{x,g} / A \quad r_{y,g} = I_{y,g} / A$$

